B.Sc. Physics. Part-III

Mathematical Physics.
(Paper-V, Group-A)

Q. Separate $\tan^{-1}(x+iy)$ into real and imaginary parts.

Solution:

Let us assume $a+ib = \tan^{-1}(x+iy)$ — ①

Then we can write $a-ib = \tan^{-1}(x-iy)$ — ②

Now adding eqs. ① and ② we obtain.

$$2a = \tan^{-1}(x+iy) + \tan^{-1}(x-iy)$$

$$= \tan^{-1}\frac{(x+iy)+(x-iy)}{1-(x+iy)(x-iy)} = \tan^{-1}\frac{(2x)}{1-(x^2+y^2)}$$

or $$\boxed{a = \frac{1}{2}\tan^{-1}\frac{2x}{1-(x^2+y^2)}} \quad \text{— ③}$$

Again Substracting eqs-① & ② we obtain.

$$2ib = \tan^{-1}(x+iy) - \tan^{-1}(x-iy)$$

$$= \tan^{-1}\frac{(x+iy)-(x-iy)}{1+(x+iy)(x-iy)}$$

$$= \tan^{-1}\frac{2iy}{1+(x^2+y^2)} = \tan^{-1} i\,\frac{2y}{1+(x^2+y^2)}$$

$$= i\tanh^{-1}\frac{2y}{1+(x^2+y^2)} \qquad \left\{\because \tan^{-1} iz = i\tanh^{-1} z\right\}$$

or $$\boxed{b = \frac{1}{2}\tanh^{-1}\frac{2y}{1+(x^2+y^2)}} \quad \text{— ④}$$

Eqs. ③ and ④ represent the corresponding real and imaginary parts.